Darboux-type Barrier Certificates for Safety Verification of Nonlinear Hybrid Systems

Wang Lin

Academy of Mathematics and System Science, CAS

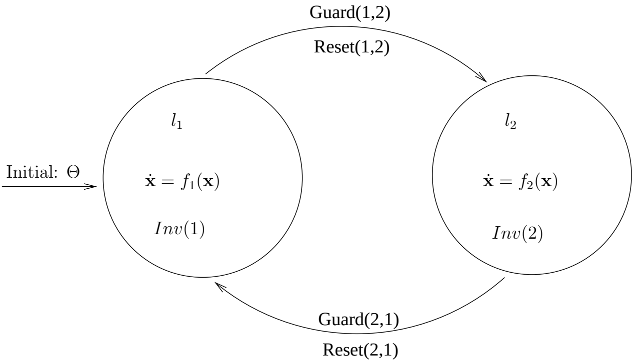
Joint work with Xia Zeng, Zhengfeng Yang, Xin Chen and Lilei Wang

Hybrid Systems

Hybrid System = Discrete Transitions + Continuous Evolutions

Hybrid Systems

Hybrid System = Discrete Transitions + Continuous Evolutions

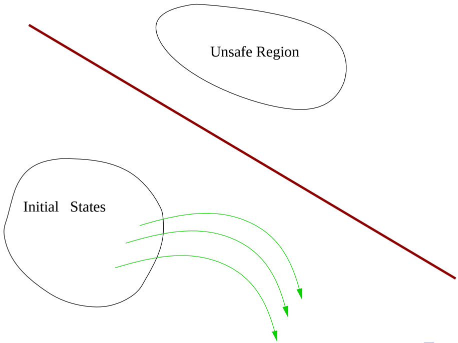


Safety Verification of Hybrid Systems

Given a hybrid system $\mathbf{H}$ with:

- an initial region $\Theta \subseteq R^n$,
- an unsafe region $X_u \subseteq R^n$.

Problem: Verify whether all trajectories of $\mathbf{H}$ that start at arbitrary $\mathbf{x}_0 \in \Theta$, can not enter the unsafe region X_u.



Verification approaches

- Reachability Analysis:
  - Exact reachable set: Quantifier elimination.

    [G. Lafferriere et al. ’00, ’01; A. Tiwari et al. ’03]
  - Approximate reachable set: ellipsoidal, polyhedra, support function, level set...

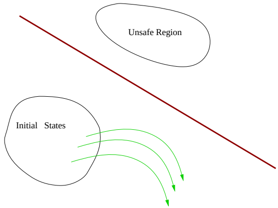
    [A. Kurzhanski and P. Varaiya ’00; T. Dang et al. ’11,’12’,13; C. Guernic and A. Girard ’09; C.J. Tomlin ’03; D. Henrion and M. Korda ’14]
- Deductive Verification:
  - Invariant-based method

    [A. Tiwari ’08,’11; A. Platzer ’09; S. Sankaranarayanan ’08, ’10; N. Zhan et al. ’11; Z. Yang et al. ’15]
  - Barrier certificate method

    [S. Prajna et al. ’04,’06,’07; H. Kong et al. ’13; L. Dai et al. 16’]

Barrier Certificates for Safety Verification

Idea: construct a barrier between the safe and unsafe areas:

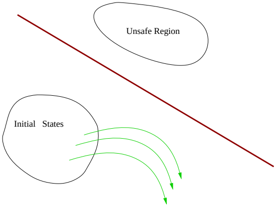


Challenges:

- How to define a new barrier certificate from different verification conditions
- How to compute barrier certificates efficiently

Barrier Certificates for Safety Verification

Idea: construct a barrier between the safe and unsafe areas:



Challenges:

- How to define a new barrier certificate from different verification conditions
- How to compute barrier certificates efficiently

- This work: Generate the new barrier certificate based on Darboux polynomial.

Darboux Polynomial

Definition (Lie Derivative)

For a vector field $F : \langle f_1, \cdots, f_n \rangle$, the Lie derivative of a smooth function $p(\mathbf{x})$ is given by

$$\mathcal{L}_F(p) = (\nabla p) \cdot F = \sum_{i=1}^{n} (\frac{\partial p}{\partial x_i} \cdot f_i).$$

Definition (Darboux polynomial)

A polynomial $p \in \mathbb{R}[\mathbf{x}]$ is called a Darboux polynomial w.r.t F if and only if

$$\mathcal{L}_F(p) = c \cdot p, \quad c \in \mathbb{R}[\mathbf{x}].$$

$p(\mathbf{x})$ is known as a first integral if $c \equiv 0$.

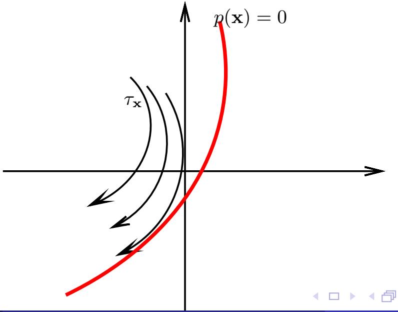
Darboux Polynomial

Key Property

$$\mathcal{L}_F(p) = c \cdot p$$

$$\Downarrow$$

$$p(\mathbf{x}_0) \geq 0 \Rightarrow p(\mathbf{x}(t)) \geq 0, \quad t > 0$$



Define Darboux-type Barrier Certificates

Given a continuous system **S** with the unsafe region X_u, the barrier certificate of S is a function $p(\mathbf{x})$ satisfying

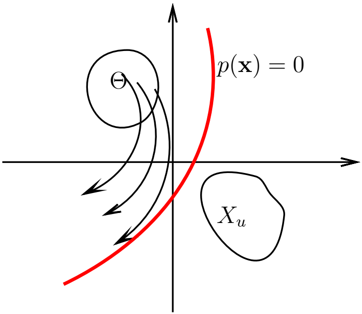
- ◇ $p(\mathbf{x})$ is a Darboux polyn.
- ◇ $\Theta \models p(\mathbf{x}) \geq 0$
- ◇ $X_u \models p(\mathbf{x}) < 0$

Define Darboux-type Barrier Certificates

Given a continuous system **S** with the unsafe region X_u, the barrier certificate of S is a function $p(\mathbf{x})$ satisfying

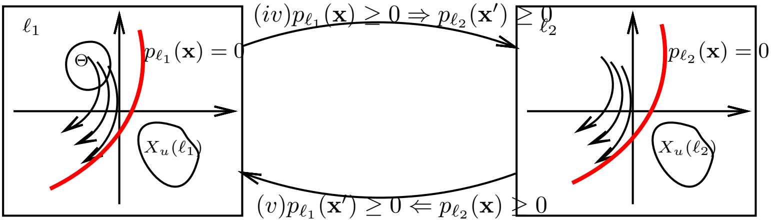
- $p(\mathbf{x})$ is a Darboux polyn.
- $\Theta \models p(\mathbf{x}) \geq 0$
- $X_u \models p(\mathbf{x}) < 0$

$\Longrightarrow$



Define Darboux-type Barrier Certificates

Given a hybrid system H with an unsafe region $X_u(\ell)$, the barrier certificate of H is a set of functions $\{p_\ell(\mathbf{x})\}$, which satisfying



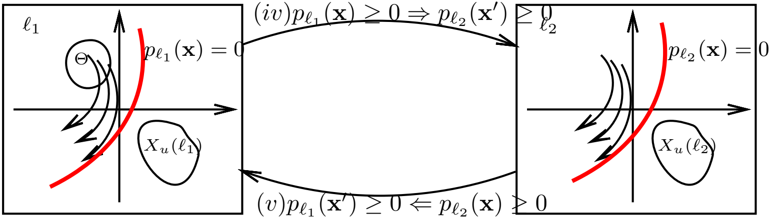
$(i) p_{\ell_1}(\mathbf{x}) \geq 0, \ \forall \mathbf{x} \in \Theta$

$(ii) p_{\ell_1}(\mathbf{x}) < 0, \ \forall \mathbf{x} \in X_u(\ell_1)$

$(iii) p_{\ell_2}(\mathbf{x}) < 0, \ \forall \mathbf{x} \in X_u(\ell_2)$

Define Darboux-type Barrier Certificates

Given a hybrid system H with an unsafe region $X_u(\ell)$, the barrier certificate of H is a set of functions $\{p_\ell(\mathbf{x})\}$, which satisfying



$(i) p_{\ell_1}(\mathbf{x}) \geq 0, \ \forall \mathbf{x} \in \Theta$

$(iii) p_{\ell_2}(\mathbf{x}) < 0, \ \forall \mathbf{x} \in X_u(\ell_2)$

$(ii) p_{\ell_1}(\mathbf{x}) < 0, \ \forall \mathbf{x} \in X_u(\ell_1)$

(iv) $p_{\ell_1}(\mathbf{x}) \geq 0 \Rightarrow p_{\ell_2}(\mathbf{x}') \geq 0 \xrightarrow{relax} \exists \lambda_{\ell_1,\ell_2} \geq 0 : p_{\ell_2}(\mathbf{x}') - \lambda_{\ell_1,\ell_2} p_{\ell_1}(\mathbf{x}) \geq 0$

(v) $p_{\ell_1}(\mathbf{x}') \geq 0 \Rightarrow p_{\ell_2}(\mathbf{x}) \geq 0 \xrightarrow{relax} \exists \lambda_{\ell_2,\ell_1} \geq 0 : p_{\ell_1}(\mathbf{x}') - \lambda_{\ell_2,\ell_1} p_{\ell_2}(\mathbf{x}) \geq 0$

Safety Verification of Continuous Systems

- ◇ $p(\mathbf{x})$ is a Darboux polyn.
- ◇ $\Theta \models p(\mathbf{x}) \geq 0$
- ◇ $X_u \models p(\mathbf{x}) < 0$

$$\Downarrow$$

$$\begin{aligned}
\text{find } & p(\mathbf{x}), c(\mathbf{x}) \in \mathbb{R}[\mathbf{x}], \\
\text{s.t. } \quad & \mathcal{L}_F(p(\mathbf{x})) = c(\mathbf{x})p(\mathbf{x}), \\
& p(\mathbf{x}) \geq 0, \quad \forall \mathbf{x} \in \Theta, \\
& p(\mathbf{x}) < 0, \quad \forall \mathbf{x} \in X_u.
\end{aligned}$$

Computing Darboux polynomial

Let $p(\mathbf{x}) = \mathbf{p}^T \cdot T_p(\mathbf{x})$, $c(\mathbf{x}) = \mathbf{c}^T \cdot T_c(\mathbf{x})$,

find $p(\mathbf{x}), c(\mathbf{x}) \in \mathbb{R}[\mathbf{x}]$, s.t. $\mathcal{L}_F(p(\mathbf{x})) = c(\mathbf{x})p(\mathbf{x})$, $p(\mathbf{x}) \geq 0, \quad \forall \mathbf{x} \in \Theta$, $p(\mathbf{x}) < 0, \quad \forall \mathbf{x} \in X_u$.	$\Longleftrightarrow$	find $\mathbf{p}, \mathbf{c}$ s.t. $F(\mathbf{p}, \mathbf{c}) = 0$, $p(\mathbf{x}, \mathbf{p}) \geq 0, \quad \forall \mathbf{x} \in \Theta$, $p(\mathbf{x}, \mathbf{p}) < 0, \quad \forall \mathbf{x} \in X_u$.

where $F(\mathbf{p}, \mathbf{c}) = [\mathbf{p}^T \cdot M_1 \cdot \mathbf{c} = 0, \cdots, \mathbf{p}^T \cdot M_k \cdot \mathbf{c}]^T$, M_i is a numeric matrix.

Computing Darboux polynomial

Let $p(\mathbf{x}) = \mathbf{p}^T \cdot T_p(\mathbf{x})$, $c(\mathbf{x}) = \mathbf{c}^T \cdot T_c(\mathbf{x})$,

$$\begin{array}{ll} \text{find } p(\mathbf{x}), c(\mathbf{x}) \in \mathbb{R}[\mathbf{x}], & \\ \quad \text{s.t.} \quad \mathcal{L}_F(p(\mathbf{x})) = c(\mathbf{x})p(\mathbf{x}), & \\ \quad\quad p(\mathbf{x}) \geq 0, \quad \forall \mathbf{x} \in \Theta, & \\ \quad\quad p(\mathbf{x}) < 0, \quad \forall \mathbf{x} \in X_u. & \end{array} \iff \begin{array}{ll} \text{find } \mathbf{p}, \mathbf{c} & \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = 0, & \\ \quad\quad p(\mathbf{x}, \mathbf{p}) \geq 0, \quad \forall \mathbf{x} \in \Theta, & \\ \quad\quad p(\mathbf{x}, \mathbf{p}) < 0, \quad \forall \mathbf{x} \in X_u. & \end{array}$$

where $F(\mathbf{p}, \mathbf{c}) = [\mathbf{p}^T \cdot M_1 \cdot \mathbf{c} = 0, \cdots, \mathbf{p}^T \cdot M_k \cdot \mathbf{c}]^T$, M_i is a numeric matrix.

$\boxed{F(\mathbf{p}, \mathbf{c}) = 0} \longrightarrow$ a quadratic system

$$\boxed{\begin{array}{ll} p(\mathbf{x}, \mathbf{p}) \geq 0, & \forall \mathbf{x} \in \Theta \\ p(\mathbf{x}, \mathbf{p}) < 0, & \forall \mathbf{x} \in X_u \end{array}} \longrightarrow \text{universal quantifier constraints}$$

- Challenge: solving semi-algebraic systems with parameters is hard!

Computing Darboux polynomial

Let $p(\mathbf{x}) = \mathbf{p}^T \cdot T_p(\mathbf{x})$, $c(\mathbf{x}) = \mathbf{c}^T \cdot T_c(\mathbf{x})$,

$$\begin{array}{ll} \text{find } p(\mathbf{x}), c(\mathbf{x}) \in \mathbb{R}[\mathbf{x}], & \\ \quad \text{s.t.} \quad \mathcal{L}_F(p(\mathbf{x})) = c(\mathbf{x})p(\mathbf{x}), & \\ \qquad p(\mathbf{x}) \geq 0, & \forall \mathbf{x} \in \Theta, \\ \qquad p(\mathbf{x}) < 0, & \forall \mathbf{x} \in X_u. \end{array} \Longleftrightarrow \begin{array}{ll} \text{find } \mathbf{p}, \mathbf{c} & \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = 0, & \\ \qquad p(\mathbf{x}, \mathbf{p}) \geq 0, & \forall \mathbf{x} \in \Theta, \\ \qquad p(\mathbf{x}, \mathbf{p}) < 0, & \forall \mathbf{x} \in X_u. \end{array}$$

where $F(\mathbf{p}, \mathbf{c}) = [\mathbf{p}^T \cdot M_1 \cdot \mathbf{c} = 0, \cdots, \mathbf{p}^T \cdot M_k \cdot \mathbf{c}]^T$, M_i is a numeric matrix.

$\boxed{F(\mathbf{p}, \mathbf{c}) = 0} \longrightarrow$ a quadratic system

$$\boxed{\begin{array}{ll} p(\mathbf{x}, \mathbf{p}) \geq 0, & \forall \mathbf{x} \in \Theta \\ p(\mathbf{x}, \mathbf{p}) < 0, & \forall \mathbf{x} \in X_u \end{array}} \longrightarrow \text{universal quantifier constraints}$$

- Challenge: solving semi-algebraic systems with parameters is hard!
- Our aim: reduce computational complexity!

Computing Darboux polynomial

Example 1: Consider the following continuous system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + 2x_1^2 x_2 \\ -x_2 \end{bmatrix},$$

Initial set: $\Theta = \{\mathbf{x} \in \mathbb{R}^2 : 1 \le x_1 \le 2 \wedge -2 \le x_2 \le -1\}$.
Unsafe region: $X_u = \{\mathbf{x} \in \mathbb{R}^2 : 1 \le x_1 \le 2 \wedge 1 \le x_2 \le 2\}$.

Computing Darboux polynomial

Example 1: Consider the following continuous system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1 + 2x_1^2 x_2 \\ -x_2 \end{bmatrix},$$

Initial set: $\Theta = \{\mathbf{x} \in \mathbb{R}^2 : 1 \le x_1 \le 2 \wedge -2 \le x_2 \le -1\}$.
Unsafe region: $X_u = \{\mathbf{x} \in \mathbb{R}^2 : 1 \le x_1 \le 2 \wedge 1 \le x_2 \le 2\}$.
Let $p = p_1 x_1 + p_2 x_2 + p_3$, and
$c = c_1 x_1^2 + c_2 x_1 x_2 + c_3 x_2^2 + c_4 x_1 + c_5 x_2 + c_6$.

$$\left.\begin{array}{ll} \text{find } \mathbf{p}, \mathbf{c} & \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = 0, & \\ \quad\quad p(\mathbf{x}, \mathbf{p}) \ge 0, & \forall \mathbf{x} \in \Theta, \\ \quad\quad p(\mathbf{x}, \mathbf{p}) < 0, & \forall \mathbf{x} \in X_u. \end{array}\right\}, \quad \text{where } F = \begin{bmatrix} 2p_1 - c_1 p_2 - c_2 p_1 \\ -c_2 p_3 - c_4 p_2 - c_5 p_1 \\ \vdots \\ -p_2 - c_5 p_3 - c_6 p_2 \end{bmatrix}$$

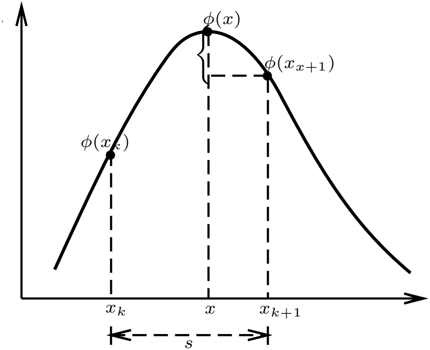
The optimization problem for computing Darboux-type barrier certificate:

var=9, # eq =11, # ctr=2

Computing Darboux polynomial

Our idea: Sampling based universal quantifier elimination

Example: Given $\Omega = [a, b]$, $\phi(x) \in \mathbb{R}[x]$, prove $\phi(x) \geq 0 \ \forall x \in \Omega$.



- For all $x \in \Omega$, $|\phi(x) - \phi(x_k)| \leq \frac{1}{2} ns\eta \triangleq \delta$, $\eta = \sup_{x \in \Omega} \|\nabla \phi(x)\|_\infty$.
- $\phi(x) \geq 0, \ \forall x \in \Omega \xrightarrow{relax} \phi(x_i) \geq \delta, \ \forall x_i \in \chi, \chi = \{x_1, x_2, \cdots, x_m\}$.
- $s \to 0 \Rightarrow \delta \to 0$.
- $\phi(x) \geq 0, \ \forall x \in \Omega \xrightarrow{approx} \phi(x_i) \geq 0, \ \forall x_i \in \chi$(with small s).

Computing Darboux polynomial

$$\left.\begin{array}{ll} \text{find } \mathbf{p}, \mathbf{c} & \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, & \\ \qquad p(\mathbf{x}, \mathbf{p}) \geq 0, & \forall \mathbf{x} \in \Theta, \\ \qquad p(\mathbf{x}, \mathbf{p}) < 0, & \forall \mathbf{x} \in X_u. \end{array}\right\}$$

↓ approximate

$$\left.\begin{array}{ll} \text{find } \mathbf{p}, \mathbf{c} & \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, & \\ \qquad p(\mathbf{x}_{1,i}, \mathbf{p}) \geq b_{1,i}, & \forall \mathbf{x}_{1,i} \in \chi_1, \\ \qquad p(\mathbf{x}_{2,i}, \mathbf{p}) \leq b_{2,i}, & \forall \mathbf{x}_{2,i} \in \chi_2. \end{array}\right\}$$

Computing Darboux polynomial

$$\left.\begin{array}{lll} \text{find } \mathbf{p}, \mathbf{c} & & \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, & & \\ \qquad p(\mathbf{x}, \mathbf{p}) \geq 0, & \forall \mathbf{x} \in \Theta, \\ \qquad p(\mathbf{x}, \mathbf{p}) < 0, & \forall \mathbf{x} \in X_u. \end{array}\right\}$$

$\Big\downarrow$ approximate

$$\left.\begin{array}{lll} \text{find } \mathbf{p}, \mathbf{c} & & \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, & & \\ \qquad p(\mathbf{x}_{1,i}, \mathbf{p}) \geq b_{1,i}, & \forall \mathbf{x}_{1,i} \in \chi_1, \\ \qquad p(\mathbf{x}_{2,i}, \mathbf{p}) \leq b_{2,i}, & \forall \mathbf{x}_{2,i} \in \chi_2. \end{array}\right\}$$

$$\Updownarrow$$

$$\left.\begin{array}{l} \text{find } \mathbf{p}, \mathbf{c} \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, \\ \qquad A \cdot \mathbf{p} \geq \mathbf{b} \end{array}\right\}$$

Computing Darboux polynomial

$$\left.\begin{array}{lll} \text{find } \mathbf{p}, \mathbf{c} & & \\ \quad \text{s.t.} & F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, & \\ & p(\mathbf{x}, \mathbf{p}) \geq 0, & \forall \mathbf{x} \in \Theta, \\ & p(\mathbf{x}, \mathbf{p}) < 0, & \forall \mathbf{x} \in X_u. \end{array}\right\}$$

$\Downarrow$ approximate

$$\left.\begin{array}{lll} \text{find } \mathbf{p}, \mathbf{c} & & \\ \quad \text{s.t.} & F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, & \\ & p(\mathbf{x}_{1,i}, \mathbf{p}) \geq b_{1,i}, & \forall \mathbf{x}_{1,i} \in \chi_1, \\ & p(\mathbf{x}_{2,i}, \mathbf{p}) \leq b_{2,i}, & \forall \mathbf{x}_{2,i} \in \chi_2. \end{array}\right\}$$

$$\Updownarrow$$

$$\left.\begin{array}{ll} \text{find } \mathbf{p}, \mathbf{c} & \\ \quad \text{s.t.} & F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, \\ & A \cdot \mathbf{p} \geq \mathbf{b} \end{array}\right\}$$

[c.f. interior point method, trust-region method...]

Computing Darboux polynomial

$$\left.\begin{array}{l}\text{find } \mathbf{p}, \mathbf{c} \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, \\ \qquad\quad p(\mathbf{x}, \mathbf{p}) \geq 0, \qquad \forall \mathbf{x} \in \Theta, \\ \qquad\quad p(\mathbf{x}, \mathbf{p}) < 0, \qquad \forall \mathbf{x} \in X_u.\end{array}\right\}$$

↓ approximate

$$\left.\begin{array}{l}\text{find } \mathbf{p}, \mathbf{c} \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, \\ \qquad\quad p(\mathbf{x}_{1,i}, \mathbf{p}) \geq b_{1,i}, \qquad \forall \mathbf{x}_{1,i} \in \chi_1, \\ \qquad\quad p(\mathbf{x}_{2,i}, \mathbf{p}) \leq b_{2,i}, \qquad \forall \mathbf{x}_{2,i} \in \chi_2.\end{array}\right\}$$

$$\Updownarrow$$

$$\left.\begin{array}{l}\text{find } \mathbf{p}, \mathbf{c} \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, \\ \qquad\quad A \cdot \mathbf{p} \geq \mathbf{b}\end{array}\right\}$$

[c.f. interior point method, trust-region method...]

Challenge: The solutions highly depend on the initial values!

Computing Darboux polynomial

Example 1 (cont.)

$$\left.\begin{aligned} &\text{find } \mathbf{p}, \mathbf{c} \\ &\qquad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, \\ &\qquad\qquad A \cdot \mathbf{p} \geq \mathbf{b}, \end{aligned}\right\}$$

where $\mathbf{p} = (p_1, p_2, p_3)^T$, $\mathbf{c} = (c_1, c_2, \cdots, c_6)^T$, and $A \in \mathbb{R}^{18\times 3}$.

Computing Darboux polynomial

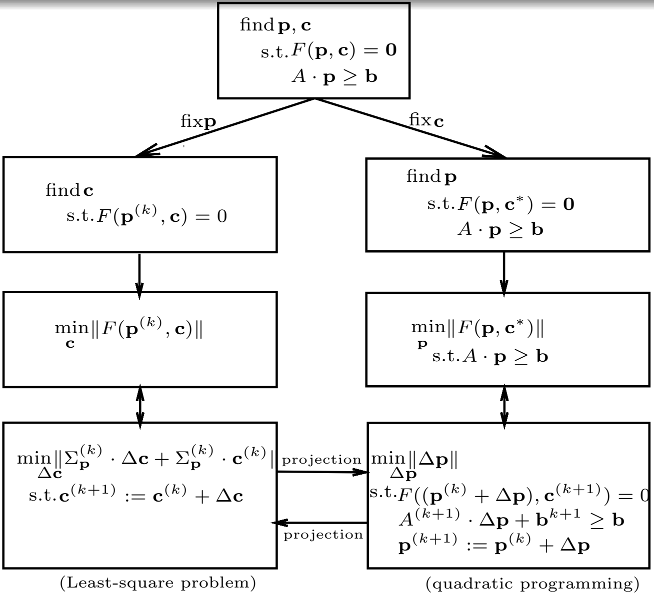
Example 1 (cont.)

$$\left.\begin{array}{l}\text{find } \mathbf{p}, \mathbf{c} \\ \quad \text{s.t.} \quad F(\mathbf{p}, \mathbf{c}) = \mathbf{0}, \\ \qquad\qquad A \cdot \mathbf{p} \geq \mathbf{b}, \end{array}\right\}$$

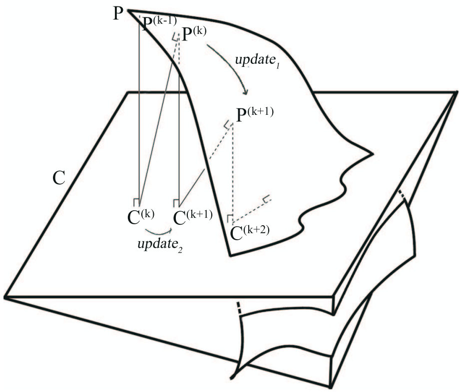
where $\mathbf{p} = (p_1, p_2, p_3)^T$, $\mathbf{c} = (c_1, c_2, \cdots, c_6)^T$, and $A \in \mathbb{R}^{18\times 3}$.

- Select the initial solution $\mathbf{p}^0$ from the set $[-5, 5]^3$. Call the Matlab Command fmincon based on interior-point algorithm.
- After 5000 trials by choosing $\mathbf{p}^0$ randomly, fmincon can not yield any feasible solution.

LS-QP Alternating Projection



LS-QP alternating projection method



P : the manifold defined by $F(\mathbf{p}, \mathbf{c}) = 0$.

C : the polyhedron defined by $A \cdot \mathbf{p} \geq \mathbf{b}$.

Example 2

Consider the polynomial system

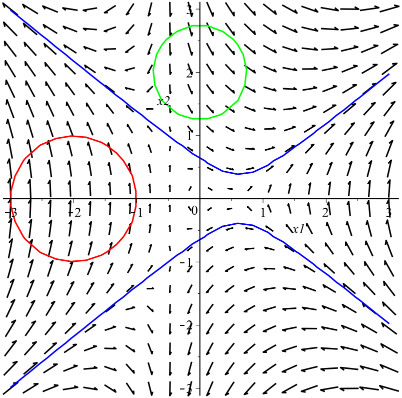
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 + 2x_1x_2 \\ -x_1 + 2x_1^2 - x_2^2 \end{bmatrix},$$

- Initial set: $\Theta = \{\mathbf{x} \in \mathbb{R}^2 : x_1^2 + (x_2 - 2)^2 \leq \frac{9}{16}\}$.
- Unsafe region: $X_u = \{\mathbf{x} \in \mathbb{R}^2 : (x_1 + 2)^2 + x_2^2 - 1\}$.

Example 2 (cont.)

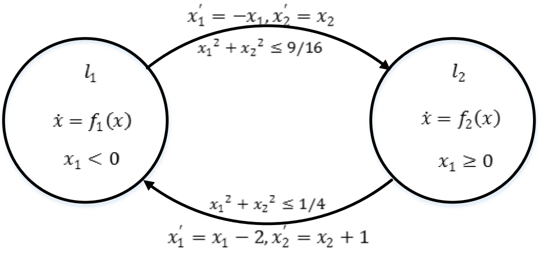
By applying our suggested method, we can obtain the following Darboux-type barrier certificate:

$$p(\mathbf{x}) = 1 - 2x_1 + \frac{8}{5}x_1^2 - \frac{12}{5}x_2^2.$$



Example 3

Consider the the following hybrid system:



where

$$f_1(\mathbf{x}) = \begin{bmatrix} -x_1 + x_1x_2 \\ -x_2 \end{bmatrix}, \quad f_2(\mathbf{x}) = \begin{bmatrix} -x_1 + 2x_1^2x_2 \\ -x_2 \end{bmatrix}$$

Initial state: $\Theta = \{(x_1, x_2) \in \mathbb{R}^2 : (x_1+2)^2 + (x_2-2)^2 \leq 0.25\}$.
Unsafe area: $X_u(\ell_2) = \{(x_1, x_2) \in \mathbb{R}^2 : (x_1-2)^2 + (x_2-2)^2 \leq 0.25\}$.

Example 3 (cont.)

By applying our method, we can obtain Darboux-type barrier certificates at locations ℓ_1 and ℓ_2:

$$p_{\ell_1}(\mathbf{x}) = 0.7332\,x_2,$$
$$p_{\ell_2}(\mathbf{x}) = -0.2711\,x_1x_2 + 0.2711,$$

which can verify the safety of the system.

Experiments: Algorithm Performance on Benchmarks

ID	n	deg	LMI		BMI		LS-QP	
			deg(B)	$T(s)$	deg(B)	$T(s)$	deg(p)	$T(s)$
1	2	2	*Fail*	—	2	0.8744	2	0.650
2	3	2	*Fail*	_	*Fail*	_	1	0.485
3	3	2	1	0.3751	1	1.1039	1	0.515
4	2	2	*Fail*	_	*Fail*	_	1	0.364
5	3	2	*Fail*	-	2	5.0523	2	1.633
6	2	2	*Fail*	_	*Fail*	_	1	0.315
7	2	2	2	0.2816	2	1.1407	1	0.371
8	2	2	*Fail*	—	2	1.9511	2	0.923
9	2	3	4	0.4623	2	1.0011	*Fail*	_
10	4	2	*Fail*	—	2	10.8846	1	7.050
11	6	2	2	27.2306	2	65.3519	1	14.06

Summary

Summary:

- Construct the Darboux-type barrier certificates for safety verification of hybrid systems
- Present an LS-QP alternating projection method to compute the Darboux-type barrier certificates.

Summary

Summary:

- Construct the Darboux-type barrier certificates for safety verification of hybrid systems
- Present an LS-QP alternating projection method to compute the Darboux-type barrier certificates.

Future Work:

- Deal with non-polynomial hybrid systems ($\sin, \cos, \exp, \ln, \cdots$).

Thank you!